ÉRETTSÉGI VIZSGA • 2022. október 18.

MATEMATIKA ANGOL NYELVEN

EMELT SZINTŰ ÍRÁSBELI VIZSGA

minden vizsgázó számára

JAVÍTÁSI-ÉRTÉKELÉSI ÚTMUTATÓ

OKTATÁSI HIVATAL

Instructions to examiners

Formal requirements:

1. Mark the paper **legibly, in ink, different in colour** from that used by the candidate.
2. The first of the rectangles next to each problem shows the maximum attainable score on that problem. The **score** given by the examiner is to be entered **into the rectangle** next to this.
3. **If the solution is perfect**, enter maximum score and, with a checkmark, indicate that you have seen each item leading to the solution and consider them correct.
4. If the solution is incomplete or incorrect, please **mark the error** and also indicate the individual **partial scores**. It is also acceptable to indicate the points lost by the candidate if it makes grading the paper easier. After correcting the paper, it must be clear about every part of the solution whether that part is correct, incorrect or unnecessary.
5. Please, **use the following symbols** when correcting the paper:
   - correct calculation: *checkmark*
   - principal error: *double underline*
   - calculation error or other, not principal, error: *single underline*
   - correct calculation with erroneous initial data: *dashed checkmark or crossed checkmark*
   - incomplete reasoning, incomplete list, or other missing part: *missing part symbol*
   - unintelligible part: *question mark* and/or *wave*
6. Do not assess anything written **in pencil**, except for diagrams

Assessment of content:

1. The answer key may contain more than one solution for some of the problems. If the **solution given by the candidate is different**, allocate the points by identifying parts of the solution equivalent to those given in the answer key.
2. Subtotals may be **further divided, unless stated otherwise in the answer key**. However, scores awarded must always be whole numbers.
3. If there is a **calculation error** or inaccuracy in the solution, take points off only for the part where the error occurs. If the reasoning remains correct and the error is carried forward while the nature of the problem remains unchanged, points for the rest of the solution must be awarded.
4. **In case of a principal error**, no points should be awarded at all for that section of the solution, not even for steps that are formally correct. (These logical sections of the solutions are separated by double lines in the answer key.) However, if the erroneous information obtained through principal error is carried forward to the next section or the next part of the problem, and it is used there correctly, the maximum score is due for that part, provided that the error has not changed the nature of the task to be completed.
5. Where the answer key shows a **unit** or a **remark** in brackets, the solution should be considered complete without that unit or remark as well.

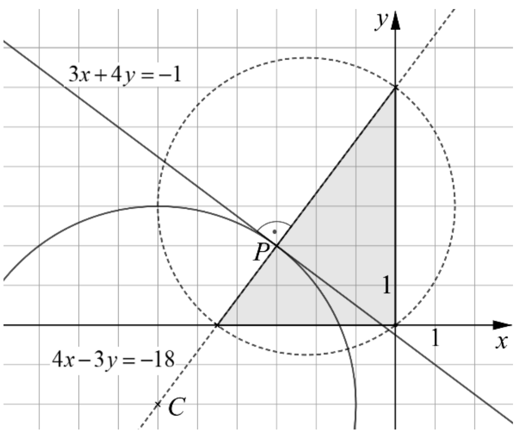
6. If there are more than one different approach to a problem, **assess only the one indicated by the candidate**. Please, mark clearly which attempt was assessed.
7. **Do not give extra points** (i.e. more than the score due for the problem or part of problem).
8. The score given for the solution of a problem, or part of a problem, **may never be negative**.
9. **Do not take points off** for steps or calculations that contain errors but are not actually used by the candidate in the solution of the problem.
10. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations**: addition, subtraction, multiplication, division, calculating powers and roots, $n!$, $\binom{n}{k}$, replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers π and e, finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
11. Using **diagrams** to justify a solution (e.g. reading data off diagrams by way of direct measurement) is unacceptable.
12. **Probabilities** may also be given in percentage form (unless stated otherwise in the text of the problem).
13. If the text of the problem does not instruct the candidate to round their solution in a particular way, any solution **rounded reasonably and correctly** is acceptable even if it is different from the one given in the answer key.
14. **Assess only four out of the five problems in part II of this paper**. The candidate was requested to indicate in the appropriate square the number of the problem not to be assessed and counted towards their total score. Should there be a solution to that problem, it does not need to be marked. However, if the candidate did not make a selection and neither is their choice clear from the paper, assume automatically that it is the last problem in the examination paper that is not to be assessed.

I.

1. a)		
The radius of the circle is $\left\|\overrightarrow{CP}\right\| = \sqrt{3^2+4^2} = 5$ units,	1 point	
the equation is $(x+6)^2+(y+2)^2=25$.	1 point	
Total:	**2 points**	

1. b)		
The tangent is perpendicular to the radius drawn to the point of tangency,	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
so a possible normal vector of the tangent is $\overrightarrow{CP}(3;4)$	1 point	
Its equation is $3x + 4y = -1$.	1 point	
Total:	**3 points**	

1. c)		
One direction vector of the line CP is $\overrightarrow{CP}(3;4)$,	1 point	
Its equation is $4x - 3y = -18$.	1 point	
(Substituting $y = 0$ and $x = 0$ yields that) the line crosses the x axis at (–4.5; 0) and the y axis at (0; 6).	1 point	
Use the Pythagorean theorem to determine the hypotenuse: $\sqrt{4.5^2+6^2} = 7.5$ units.	1 point	
The radius of the circumcircle of a right triangle is half of the hypotenuse,	1 point	*The area of the triangle is 13.5*
that is 3.75 units.	1 point	*Use the formula* $R=\frac{abc}{4T}$ *to determine the radius of the circumcircle:* $\frac{4.5\cdot 6\cdot 7.5}{4\cdot 13.5}=3.75$ *units.*
Total:	**6 points**	



2. a) Solution 1		
cos $x = 0$ is not a solution (as sin $x = 0$ is impossible then),	1 point	
so this equation is equivalent to $\frac{\sin^2 x}{\cos^2 x} = 3$, i.e. $\tan^2 x = 3$.	1 point	
$\tan x = \sqrt{3}$ or $\tan x = -\sqrt{3}$.	2 points	
if $\tan x = \sqrt{3}$ then $x = \frac{\pi}{3} + k\pi$, $k \in \mathbf{Z}$, if $\tan x = -\sqrt{3}$ then $x = -\frac{\pi}{3} + k\pi$, $k \in \mathbf{Z}$.	2 points	
Total:	**6 points**	

2. a) Solution 2		
Use $\sin^2 x = 1 - \cos^2 x$ for the equivalent form	1 point	
$\frac{1}{4} = \cos^2 x$.	1 point	
$\cos x = \frac{1}{2}$ or $\cos x = -\frac{1}{2}$.	2 points	
if $\cos x = \frac{1}{2}$ then $x = \pm\frac{\pi}{3} + 2k\pi$, $k \in \mathbf{Z}$, if $\cos x = -\frac{1}{2}$ then $x = \pm\frac{2\pi}{3} + 2k\pi$, $k \in \mathbf{Z}$.	2 points	$x = \frac{\pi}{3} + k\pi$ or $x = -\frac{\pi}{3} + k\pi$, $k \in \mathbf{Z}$
Total:	**6 points**	

Notes:
1. Award a maximum of 5 points if the candidate gives the correct answer in degrees.
2. Award a maximum of 4 points if the candidate does not give periods.
3. Award a maximum of 5 points if the candidate applies the correct periods but completely misses the condition $k \in \mathbf{Z}$.

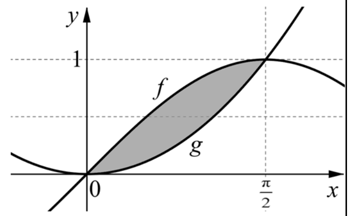
2. b)		
The domain of the equation is: $x > 2$.	1 point	*Award this point if the candidate checks by substitution.*
(As per the definition and rules of logarithms) $\log_3 \frac{(x+8)(x-2)}{x+4} = \log_3 3$.	2 points	
(As the logarithm function is a one-to-one mapping) $\frac{(x+8)(x-2)}{x+4} = 3$.	1 point	
Rearranged: $x^2 + 3x - 28 = 0$.	1 point	
The roots of this equation are 4 and –7.	1 point	
Only 4 is a correct solution (as $-7 \leq 2$).	1 point	
Check by substitution or reference to equivalent steps over the domain.	1 point	
Total:	**8 points**	

<table>
<tr><td colspan="3">3. a)</td></tr>
<tr><td>The pie chart contains data about 24 chargers, so each charger corresponds to a 15° central angle.</td><td>1 point</td><td></td></tr>
<tr><td>The table showing data about the 25 chargers:
lifespan (months) | 49 | 50 | 51 | 52 | 53 | 54
no. of chargers | 2 | 4 | 7 | 4 | 5 | 2</td><td>1 point</td><td></td></tr>
<tr><td>The mean lifespan of the 24 chargers:
$\frac{2\cdot 49+4\cdot 50+7\cdot 51+4\cdot 52+5\cdot 53+2\cdot 54}{24}=$
= 51.5 (months).</td><td>1 point</td><td></td></tr>
<tr><td>The standard deviation:
$\sqrt{\frac{2\cdot(49-51.5)^2+...+2\cdot(54-51.5)^2}{24}}=$
$=\sqrt{\frac{4\cdot 2.5^2+9\cdot 1.5^2+11\cdot 0.5^2}{24}}=$</td><td>1 point</td><td>Award this point if the candidate obtains the correct answer using a calculator.</td></tr>
<tr><td>$=\sqrt{2}$ (≈ 1.41) (months).</td><td>1 point</td><td></td></tr>
<tr><td align="right">Total:</td><td>5 points</td><td></td></tr>
</table>

<table>
<tr><td colspan="3">3. b)</td></tr>
<tr><td>The probability that a randomly selected charger has a lifespan below 50 months is (1 – 0.9 =) 0.1.</td><td>1 point</td><td>Award this point if the correct reasoning is reflected only by the solution.</td></tr>
<tr><td>(Let P(n) be the probability that there will be exactly n chargers out of 20 with a lifespan below 50 months.) $P(0)=0.9^{20}$ (≈ 0.122)</td><td>1 point</td><td></td></tr>
<tr><td>$P(1)=\binom{20}{1}\cdot 0.1\cdot 0.9^{19}$ (≈ 0.270)
$P(2)=\binom{20}{2}\cdot 0.1^2\cdot 0.9^{18}$ (≈ 0.285)</td><td>2 points</td><td></td></tr>
<tr><td>The final probability is:
$P(0)+P(1)+P(2)\approx 0.677$.</td><td>1 point</td><td></td></tr>
<tr><td align="right">Total:</td><td>5 points</td><td></td></tr>
</table>

<table>
<tr><td colspan="3">3. c)</td></tr>
<tr><td>Let p be the probability that a charger has a lifespan of at least 55 months. In this case $(1-p)^5=0.75$.</td><td>1 point</td><td></td></tr>
<tr><td>Here $1-p$ (= $\sqrt[5]{0.75}$) ≈ 0.944,</td><td>1 point</td><td></td></tr>
<tr><td>and the final probability is $p\approx 0.056$.</td><td>1 point</td><td></td></tr>
<tr><td align="right">Total:</td><td>3 points</td><td></td></tr>
</table>

4. a)		
$f(0)=\sin 0=0$, $f\left(\frac{\pi}{2}\right)=\sin\frac{\pi}{2}=1$ $g(0)=\left(\frac{0}{\pi}\right)^2=0$, $g\left(\frac{\pi}{2}\right)=\left(\frac{2}{\pi}\cdot\frac{\pi}{2}\right)^2=1$ (And so the statement is true.)	3 points	*Award 2 points for one error, 1 point for two errors.*
Total:	**3 points**	

4. b)		
(As $f(x) \geq g(x)$ for the given interval) the area of the figure is $\int_0^{\frac{\pi}{2}}\left(\sin x-\left(\frac{2x}{\pi}\right)^2\right)dx=\int_0^{\frac{\pi}{2}}\left(\sin x-\frac{4}{\pi^2}\cdot x^2\right)dx=$	1 point	
$=\left[-\cos x-\frac{4}{\pi^2}\cdot\frac{x^3}{3}\right]_0^{\frac{\pi}{2}}=$	2 points	
$=\left(-0-\frac{\pi}{6}\right)-(-1-0)=$	1 point	
$=-\frac{\pi}{6}+1$ (≈ 0.476).	1 point	
Total:	**5 points**	

4. c)		
$a_n=\frac{2}{n}+2\pi$	1 point	$a_{n+1}-a_n=\frac{-2}{n(n+1)}$ *This is always negative, so the sequence is (strictly) monotone decreasing.*
The sequence $\left\{\frac{2}{n}\right\}$ is (strictly) monotone decreasing, while 2π is a constant, which makes the whole sequence (strictly) monotone decreasing.	1 point	
The sequence is bounded from above, as the first term of a decreasing sequence is always an upper bound, too. It is also bounded from below, as all terms are positive (and so 0, for example, is a lower bound).	1 point*	*The lowest upper bound of the sequence is* $2+2\pi$, *the highest lower bound is* 2π.
The limit of the sequence $\left\{\frac{2}{n}\right\}$ is 0,	1 point	
so the limit of the given sequence is ($0+2\pi=$) 2π.	1 point	
Total:	**5 points**	

*Note: Award the point marked with * if the candidate first proves that the given sequence is convergent, then refers to how this property leads to also being bounded.*

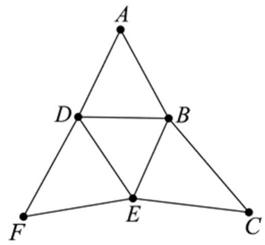
II.

5. a) Solution 1		
(Each pair of regions A, B and D are adjacent and so) region A can be coloured in 4 different ways, region B 3 ways, region D 2 ways.	1 point	
The total number of possible colourings for regions A, B, D is $4 \cdot 3 \cdot 2$ (= 24).	1 point	
Let's assume that e.g. A is red, B is blue, D is green. C must not be red or blue then (only green or yellow).	1 point	*Region E must not be blue, nor green (i.e. has to be red or yellow).*
If C is yellow, then E must be red, while F can be of two colours (blue or yellow). That makes two possible colourings for E and F.	1 point	*If E is yellow, then C is green, and F can be of two colours (red or blue).*
If C is green, then E can be of two colours (red or yellow), and F can also be of two colours in both cases (if E is red, then F is blue or yellow, if E is yellow, then F is red or blue). There are $2 \cdot 2$ possible ways for E and F.	1 point	*If E is red, then C can be of two colours (green or yellow) and F can also be of two colours (blue or yellow).*
Altogether, there are $4 \cdot 3 \cdot 2 \cdot (2 + 2 \cdot 2) =$	1 point	
= 144 possible ways to colour the diagram.	1 point	
Total:	**7 points**	

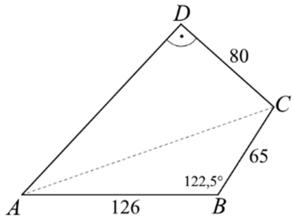
5. a) Solution 2		
(Each pair of regions A, B and D are adjacent and so) region A can be coloured in 4 different ways, region B 3 ways, region D 2 ways.	1 point	
The total number of possible colourings for regions A, B, D is $4 \cdot 3 \cdot 2$ (= 24).	1 point	
Let's assume that e.g. A is red, B is blue, D is green. In this case, E can be of two colours (red or yellow). After colouring E, there are only two options left for F (as it is adjacent to both B and E).	1 point	
If C could also be red, then (after colouring A, B, D, E and F) C could be of two colours (as it is adjacent to B and E). This makes a total $2 \cdot 2 \cdot 2$ (= 8) possible ways to continue the colouring that begins with A red, B blue, C green.	1 point	
If A is red, B blue and C green, then every colouring in which C is red is wrong. In this case, E must be yellow, and F can be of two colours (red or blue), so there are two wrong colourings.	1 point	

Subtract the number of wrong colourings (4·3·2·2) from the total (4·3·2·2·2·2) to get the number of right colourings.	1 point	
That is 4·3·2·(8 – 2) = 144.	1 point	192 – 48 = 144
Total:	**7 points**	

Note: If the regions are associated with the vertices of a graph and edges represent adjacency, then the problem can be rephrased into colouring the vertices of a graph appropriately (in any correct colouring all edges connect vertices of different colours). The appropriate graph is:



5. b)		
(Express the numbers B, E and F in terms of C.) By (1) and (2): $B = \frac{6+C}{2}$.	1 point	*By* (1) *and* (2)*: if* $B = 6 + m$*, then* $C = 6 + 2m$.
By (5): $E = C + 2$, by (3) and (1): $F = \sqrt{DE} = \sqrt{8(C+2)}$.	1 point	*also, by* (5): $E = 8 + 2m$, *and by* (4): $F = 7 + m$.
By (4): $F = B + 1$, i.e. $\sqrt{8(C+2)} = \frac{6+C}{2} + 1$.	1 point	*By* (1) *and* (3): $F = \sqrt{8(8+2m)}$, *so* $\sqrt{8(8+2m)} = 7 + m$
Apply a common denominator on the right, then square: $8(C+2) = \frac{64+16C+C^2}{4}$.	1 point	$8(8+2m) = (7+m)^2$ $64 + 16m = 49 + 14m + m^2$
Multiply by 4 and rearrange: $0 = C^2 - 16C$.	1 point	$m^2 - 2m - 15 = 0$
The roots are $C = 0$ and $C = 16$.	1 point	$m = -3$ *vagy* $m = 5$
In the first case, the numbers are: $B = 3$, $C = 0$, $E = 2$, $F = 4$.	1 point	
In the second case, the numbers are: $B = 11$, $C = 16$, $E = 18$, $F = 12$.	1 point	
Check against the text.	1 point	
Total:	**9 points**	

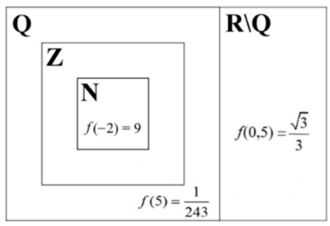
6. a)		
 The area of triangle ABC is $\frac{126 \cdot 65 \cdot \sin 122.5°}{2} \approx$ ≈ 3454 (m^2).	1 point	
Use the Law of Cosines on triangle ABC to determine the length of diagonal AC: $AC = \sqrt{126^2 + 65^2 - 2 \cdot 126 \cdot 65 \cdot \cos 122.5°} \approx 170$ (m).	1 point	
Use the Pythagorean theorem on the right triangle ADC: $AD = \sqrt{170^2 - 80^2} = 150$ (m).	1 point	
The area of triangle ADC: $\frac{AD \cdot DC}{2} = \frac{150 \cdot 80}{2} = 6000$ (m^2).	1 point	
The area of the land is (3454 + 6000 =) 9454 m^2 = 0,9454 ha.	1 point	
0.9454 : 0.9 ≈ 1.05, so the real area is about 5% more than what is advertised.	1 point	
Total:	**6 points**	

6. b)		
The shape of the mass of water that remains in the tilted trough is a regular triangle-based pyramid. The base edge is 38 cm, the height is 72 cm.	2 points	*Award these 2 points if the correct reasoning is reflected only by the solution.*
The area of the regular triangle: $A = \frac{38^2 \cdot \sqrt{3}}{4}$ (≈ 625.3) (cm^2).	1 point	
The volume of the water: $\frac{A \cdot 72}{3}$, that is about 15 006 cm^3,	1 point	
which, rounded, gives 15 litres, indeed.	1 point	
Total:	**5 points**	

Note: The volume of the trough is about 45 000 cm^3 *(2 points), the volume of the pyramid is one third of this (1 point), that is* 15 000 cm^3 *(1 point), or* 15 *litres (1 point).*

6. c) Solution 1		
These 15 litres is now the volume of a regular triangle-based right prism, whose height is 72 cm.	1 point	
The area of the triangular base is $\frac{15\,000}{72} \approx 208.3$ (cm^2).	1 point	
The triangle is regular, with sides x. $\frac{x^2 \cdot \sqrt{3}}{4} \approx 208.3$	1 point	
$x \approx 21.9$ (cm),	1 point	
The depth of the water is therefore $x \cdot \frac{\sqrt{3}}{2} \approx 19$ cm.	1 point	
Total:	**5 points**	

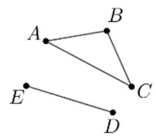
6. c) Solution 2		
The ratio of the volumes of a pyramid and a prism of the same base and height is 1 : 3, so the volume of the water still in the trough is one third of the volume of the trough.	1 point	
The base area of the prism of water is, therefore, one third of the area of the original triangle (while the height is equal).	1 point	
The bases of the prisms are similar,	1 point	
the ratio of similarity is $\frac{1}{\sqrt{3}}$.	1 point	
The depth of the water remaining in the trough is $\left(38 \cdot \frac{\sqrt{3}}{2}\right) \cdot \frac{1}{\sqrt{3}} = 19$ cm.	1 point	
Total:	**5 points**	

7. a)		
$f(-2) = 3^{-(-2)} = 9 \in \mathbf{N}$	1 point	
$f(0,5) = 3^{-\frac{1}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} \in (\mathbf{R} \setminus \mathbf{Q})$	1 point	
$f(5) = 3^{-5} = \frac{1}{3^5} = \frac{1}{243} \in (\mathbf{Q} \setminus \mathbf{Z})$	1 point	
Total:	**3 points**	

Notes:

1. Award a maximum of 2 points if the candidate does not justify their answer (does not show the calculated function values).

2. Award a maximum of 2 points if the candidate works with the function $x \mapsto 3^x$.

7. b)		
(Associate numbers with the letters of the point they are assigned to.) *A, B* and *C* are rational $\left(\frac{1}{3^2}, \frac{1}{3^7}, \frac{1}{3^{12}}\right)$. The sum of any two rational numbers is also rational, so all three edges of the subgraph *ABC* are there.	1 point	
D and *E* are irrational (The sum of two irrational numbers may be rational or irrational, too.) $E=\frac{1}{\sqrt{2}-1}=\frac{1}{\sqrt{2}-1}\cdot\frac{\sqrt{2}+1}{\sqrt{2}+1}=\sqrt{2}+1$ $D+E=1-\sqrt{2}+\sqrt{2}+1=2$, so the edge *DE* exists.	2 points	$D+E=1-\sqrt{2}+\frac{1}{\sqrt{2}-1}=$ $=\frac{(1-\sqrt{2})(\sqrt{2}-1)+1}{\sqrt{2}-1}=$ $=\frac{2\sqrt{2}-2}{\sqrt{2}-1}=\frac{2(\sqrt{2}-1)}{\sqrt{2}-1}=2$
The sum of a rational and an irrational number is always irrational, so there are no more edges in the graph.	1 point	
The graph has 4 edges.	1 point	
Total:	**5 points**	

Note: Award 2 points for a correct graph without explanation.

7. c)		
The number of rectangles is determined by the solution of the inequality $\frac{1}{3^n}-\frac{1}{3^{n+1}}>10^{-6}$.	1 point	
Multiply by 3^{n+1} (positive): $3-1>10^{-6}\cdot 3^{n+1}$, hence $2\cdot 10^6>3^{n+1}$.	1 point	
The function $x\mapsto 3^x$ ($x\in\mathbf{R}$) is (strictly) monotone increasing,	1 point	
so $\log_3(2\cdot 10^6)\approx 13.2>n+1$,	1 point	
and so the greatest positive integer solution is $n = 12$.	1 point	
One side of each of the 12 rectangles is 1. The lengths of the other sides and, thereby, the areas form the first 12 terms of a geometric sequence where $a_1=q=\frac{1}{3}$.	1 point	
$S_{12}=\frac{1}{3}\cdot\frac{\left(\frac{1}{3}\right)^{12}-1}{\frac{1}{3}-1}\approx$	1 point	
≈ 0.5, the sum of the areas of the rectangles.	1 point	
Total:	**8 points**	

8. a)		
The length of the third edge is $\left(\frac{72}{4\cdot 2}=\right)$ 9 (dm),	1 point	
so the surface area of the cuboid is $(2\cdot(4\cdot 2+4\cdot 9+2\cdot 9)=)$ 124 dm^2.	1 point	
Total:	**2 points**	

8. b)		
The edges of the cuboid (in dm) are a, $2a$ and b ($a, b > 0$).	1 point	*Award this point if the correct reasoning is reflected only by the solution.*
The volume: $72 = 2a^2b$, gives $b=\frac{36}{a^2}$.	1 point	
The surface area of the cuboid (in dm^2): $4a^2+6ab=4a^2+\frac{216}{a}$.	1 point	
The function $A:\mathbf{R}^+\to\mathbf{R};\ A(a)=4a^2+\frac{216}{a}$ is differentiable, $A'(a)=8a-\frac{216}{a^2}$.	2 points*	
Condition for the minimum is $A'(a) = 0$. Then $a = 3$.	1 point*	
As the second derivative is positive over the whole domain this is, in fact, a minimum.	1 point*	$A''(a)=8+\frac{432}{a^3}>0$
The edges of the cuboid with minimal surface area are 3 dm, 6 dm and (36 : 9 =) 4 dm long.	1 point	
Total:	**8 points**	

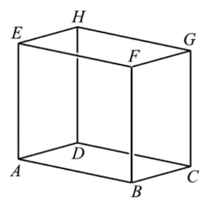
*Note: The 4 points marked * may also be awarded for the following reasoning:*

$A(a)=4a^2+\frac{216}{a}=4a^2+\frac{108}{a}+\frac{108}{a}$	1 point	
Apply the inequality between arithmetic and geometric means: $4a^2+\frac{108}{a}+\frac{108}{a}\geq 3\cdot\sqrt[3]{4a^2\cdot\frac{108}{a}\cdot\frac{108}{a}}=$ $=3\cdot\sqrt[3]{46\,656}=3\cdot 36=108$.	1 point	
Equation (i.e. minimal surface area) occurs when $4a^2=\frac{108}{a}$,	1 point	
hence $a = 3$ (dm).	1 point	

8. c) Solution 1		
There may be no vertices connected by an edge of the cuboid among those selected as, in that case, any corresponding plane (that of a face or a diagonal) will necessarily contain a fourth vertex.	2 points	
Two vertices from either the base or the top of the cuboid must necessarily be selected. These must be diagonally positioned. Such selection may be done in 2 different ways for both the base and the top face.	2 points	
The third vertex must be selected such that it will not be connected with an edge to either of the two others. This can be done in 2 different ways in each case.	1 point	
The total number of different possible selections is, therefore, $2\cdot2\cdot2 = 8$.	1 point	
Total:	**6 points**	

8. c) Solution 2		
(Counting complements:) 3 vertices out of 8 may be selected in $\binom{8}{3}=56$ different ways.	1 point	
Selecting three vertices of any face is wrong, as the fourth vertex will also be contained.	1 point	
There are 4 ways to select 3 vertices on any face and so this covers $6\cdot4 = 24$ possibilities.	2 points	
By the same reasoning: none of the 6 diagonal planes inside a cuboid are suitable to select three vertices out of the four either. This means another 24 possibilities.	1 point	
Any one of the remaining 56 – 24 – 24 = 8 possibilities (the other endpoints of the three edges from either of the 8 vertices of the cuboid) will be suitable.	1 point	
Total:	**6 points**	

Note: Award 4 points if the candidate lists the appropriate selections (ACF, ACH, BDE, BDG, EGD, FHA, FHC – the four faces of tetrahedrons ACFH and BDEG) and thereby provides the correct answer but without any further explanation.

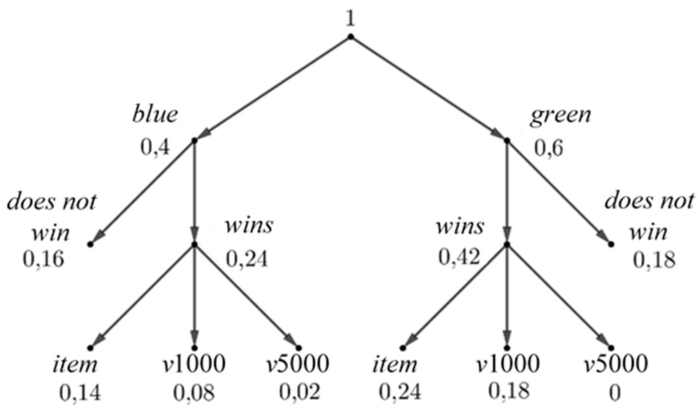


9. a)		
(Let k be the cost of a blue ticket and let z be the cost of a green one.) $5k + 3z = 6700$ and $3k + 2z = 4200$.	1 point	
Double the first equation and subtract three times the second. This gives $k = 800$, the cost of a single blue ticket.	2 points	
In this case, $z = 900$, so the cost of a single green ticket is 900 Ft.	1 point	
Check: 5 blue and 3 green tickets: 4000 + 2700 = 6700 Ft, 3 blue and 2 green tickets: 2400 + 1800 = 4200 Ft.	1 point	
Total:	**5 points**	

Note: Deduct a total of 1 point if the candidate gives their answer without units.

9. b)		
Let the number of raffle tickets be n. In this case, the number of blue tickets is $0.4n$, 35% of which will win a prize item. The probability that the ticket drawn is blue and wins a prize item is $P(AB) = \dfrac{0.4n \cdot 0.35}{n} = 0.14$.	1 point	$P(AB) = 0.4 \cdot 0.35 =$ $= 0.14$
Out of n tickets there are $0.6n$ green ones, 40% of which wins a prize item. The probability that the ticket drawn is green and wins a prize item is $\dfrac{0.6n \cdot 0.4}{n} = 0.24$.	1 point	*($\overline{B}$ is the event that a green ticket is drawn.)* $P(A\overline{B}) = 0.6 \cdot 0.4 = 0.24$
A ticket that wins a prize item is either blue or green (which are mutually exclusive) and so $P(A) = 0.14 + 0.24 = 0.38$, indeed.	1 point	$P(A) = P(AB) + P(A\overline{B}) =$ $= 0.14 + 0.24 = 0.38$
$P(B \mid A) = \dfrac{P(AB)}{P(A)} = \dfrac{0.14}{0.38} =$	1 point	
$= \dfrac{7}{19} \approx 0.368$	1 point	
(It is stated that $P(B) = 0.4$.) $P(A) \cdot P(B) = 0.38 \cdot 0.4 = 0.152$	1 point	$P(B \mid A) \neq P(B)$, *or* $P(A \mid B) \neq P(A)$
As $P(A) \cdot P(B) \neq P(AB)$,	1 point	
therefore the events A and B are not independent.	1 point	
Total:	**8 points**	

Notes:
1. Deduct a total of 1 point if the candidate arbitrarily decides the number of tickets and gives the correct answer thereby, but does not refer to how this can also be generalised.
2. The graph shows the probabilities for a particular ticket drawn.
(E.g. P(A) = 0.14 + 1.24 = 0.38.*)*



9. c)		
The expected value of a single blue ticket drawn is (0.4·0 +) 0.35·500 + 0.2·1000 + 0.05·5000 =	2 points	
= 625 Ft.	1 point	
Total:	**3 points**	